A new Stochastic Hybrid Technique for DER Problem

Mohsen Mohammadi
Department of Electrical Engineering,
Payame Noor University (PNU), Tehran, Iran

Nasser yousefi
Department of Electrical Engineering, Ardabil Branch, Islamic Azad University, Ardabil, Iran

ABSTRACT
This paper presents a new Hybrid Particle Swarm optimization with Time Varying Acceleration Coefficients (HPSOTVAC) and Bacteria Foraging Algorithm (BFA) namely (PSOTVAC/BFA) base fuzzy stochastic long term approach for determining optimum location and size of Distributed Energy Resources (DERs). The Monte Carlo simulation method is used to model the uncertainties associated with long-term load forecasting. A proper combination of several objectives is considered in the objective function. Reduction of loss and power purchased from the electricity market, loss reduction in peak load level and reduction in voltage deviation are considered simultaneously as the objective functions. At first these objectives are fuzzified and designed to be comparable with each other and then they are introduced to a PSOTVAC/BFA algorithm in order to obtain the solution which maximizes the value of integrated objective function. The output power of DERs is scheduled for each load level. An enhanced economic model is also proposed to justify investment on DER. IEEE 30-bus radial distribution test system is used as an illustrative example to show the effectiveness of the proposed method.

Key words: component; Distributed energy resources; Fuzzy optimization; Loss reduction; PSOTVAC/BFA; Voltage deviation reduction; Stochastic programming

1. Introduction
The uncertainties associated with load forecasting and equipments’ unavailability affect the system operation and planning decisions. Applying a proper method for modeling these uncertainties in planning phase, one can reduce the risk of the decisions as well as the stochastic cost of operation. Ignoring the uncertainties in planning process leads to a high risk and renders the stochastic saving gained by applying the decisions non-optimal.

In this paper a new methodology to solve the complicated problem of finding optimal location and size of Distributed Energy Resources (DERs) is presented which considers the uncertainties associated with load forecasting. In the proposed stochastic planning scheme, the stochastic characteristics of load growth are simulated using the Monte Carlo simulation method. Each possible system state is represented by a scenario. Scenario reduction technique is employed to decrease the number of created scenarios.

Particle Swarm Optimization (PSO) is one of the modern heuristic algorithms. The algorithm is based on the social interaction between search agents in feasible search space. Each particle is changing their position and velocity of each individual based on their own previous best position, and the best previous position of their neighbors. Generally, PSO is characterized as an easy concept, easy to implement, and computationally effective [1]. Unlike the other heuristic techniques, PSO has a well-balanced mechanism and flexible to amplify the global and local exploration abilities. However this is possible to occur that PSO converges junior. Recently, this method is used to various fields of power system optimization problem such as reactive power dispatch, voltage control and optimal power flow [2].

Intelligent methods are frequent techniques that can search not only local optimal solutions but also a global optimal solution depending on problem domain and execution time limit. The old optimization methods have the advantage of searching the solution space more thoroughly. The major difficulty is their sensitivity to the choice of parameters. Among intelligent methods, Particle Swarm Optimization with Time Varying Acceleration Coefficients (PSOTVAC) is strong and simple. It requires less computation time and memory. It has also standard values for its parameters.

On the other hand, Bacteria Foraging Algorithm (BFA) which is introduced by Passino [3] as a tool of optimization is a strong algorithm. In this paper, to overcome the problems of the previous techniques, the Hybrid PSOTVAC/BFA is proposed to solve proposed problem in power system. It is also seen that some simple adaptive feature incorporated in the main algorithm makes its convergence even faster. Different studies have been conducted and variety of methods has been proposed for optimal placement and parameter setting of UPFC with different objective functions in the literature. The rest of this section introduces some of the previous studies in this field and also discusses the contributions of the present work that cover the blind spots of the former studies.
Restructuring of power systems has caused an increasing interest in DERs. Successful application and worldwide tendency to DERs have led to emergence of new technologies in this area. Moreover the increasing awareness of environmental issues has more motivated the application of DERs [4].

Many benefits are gained by placement of Distributed Energy Resources (DERs), yet they may cause some troubles in operation of distribution systems if they are installed without thorough consideration. Therefore special cares should be taken in locating and sizing of DERs. A wide range of benefits, from loss reduction to voltage profile improvement, can be gained by placement of DERs in distributed systems. Therefore the realm of study of distribution systems is replete with the works on solving the problem of DER placement with different objective functions. In [5] the most important benefits of DER are modeled in economic terms. A set of indices are proposed in [6] for modeling and quantifying of the technical benefits of DERs. The rest of this section introduces some of the previous works on DER placement and also presents the contributions of the present work that cover the weaknesses of the former studies.

A. Literature review

A Distributed Generation (DG) capacity investment planning algorithm was proposed in [7] using a new heuristic approach from the perspective of a distribution company. Optimal solution for DG capacity placement was obtained through a cost-benefit analysis approach based on this new optimization model. The model aimed to minimize the disco’s operating and investment costs as well as cost of power loss, but the other benefits gained by DG placement were not considered. The analytical optimization process for determining the optimal location and size of DER was aimed at minimization of power loss of distribution systems in [8]. Both radial and meshed distribution systems were considered.

Cost-Benefit analysis is one of the other approaches used in the literature. For example, a new heuristic approach for DG capacity investment planning from the perspective of a distribution company was proposed in [9]. In order to solve the placement and sizing problem a multi-period AC optimal power flow (OPF) was proposed in [10]. Minimization of power loss was again the aim of the optimization algorithm. Loss reduction and reliability improvement were also handled in [9] as a cost/worth analysis.

Reference [11] proposed two multi-objective formulations based on the Genetic Algorithm (GA) and an ε-constrained method as optimization techniques for the placement and sizing of DER in distribution networks. The optimization process was a compromise between reduction in power losses, reliability improvement, and reduction in power to be purchased from the power market and minimization of the cost of network upgrading.

Reference [12] studied facility-location problems while taking into account a hybrid uncertain environment involving both randomness and fuzziness. Since the fuzzy parameters of the locating problem are represented in the form of continuous fuzzy variables, the determination of Value-at-Risk is inherently an infinite-dimensional optimization problem that is not possible to be solved analytically. Therefore a two-stage fuzzy facility location problem with Value-at-Risk, was proposed in [13], which results in a two-stage fuzzy zero-one integer programming problem. Both the costs and demands are skillfully assumed to be fuzzy random variables in [14], a Value-at-Risk based fuzzy facility location model is built and a hybrid modified PSO approach is proposed to solve such complicated problem.

A fuzzified multi-objective GA based algorithm was proposed in [15] for capacitor placement. Though the objective was finding the best location of capacitors in distribution networks, the model of objective function and the methodology can be used in DG placement problem. Reference [16] proposed a method for reliability improvement and loss reduction by installing fixed capacitor in a distribution system. Though the problem was the capacitor placement, the reduction of power loss at peak load was skillfully modeled as one of the objective functions.

Two types of load uncertainties for planning studies can be identified in power systems’ planning, uncertainties associated with load forecasting and short-term uncertainties related to time/weather factors. Both of them were considered in [17] and a simple GA-based optimization algorithm was used to extract the best location and size of DERs in a distribution system. Monte-Carlo simulation was used to model the stochastic nature of the system loads. The stochastic nature of system components and load growth forecasting was simulated and each possible system state was represented by a scenario in [18]. A scenario reduction technique was used to decrease the computational burden of large number of scenarios.

B. Motivations and contributions

1. Developing a long term stochastic load model for DER placement considering stochastic load growth: A long term stochastic model for system uncertainties is presented in this paper that is suited for application along with PSOTVAC/BFA algorithm. The results of case studies show the necessity of stochastic modeling of the problem. Some other studies in the literature have considered the stochastic nature of the load and system components, but uncertainties are modeled in just one hour or just one year. In planning problems, it is necessary to model the uncertainties in the entire planning horizon.

2. Considering system operation in planning phase for different system states: In this paper the output power is scheduled for each load level to avoid the inconvenient rejection of more optimal solutions. In contrast previous works considered the output power of DERs to be fixed at the maximum rated value while the load varies at each bus. This may render some optimal solutions infeasible due to violation of some constraints such as voltage magnitude limits in some load levels while in most of the other load levels there is no violation.

3. Application of fuzzy optimization approach to satisfy different objectives simultaneously in DER placement: So many studies have been conducted to reduce the cost of loss in distribution systems. Reduction of voltage deviation in order to reach a more flat voltage profile has also been the subject of many studies in distribution systems. In this paper the reduction of loss and power purchased from the electricity market, loss reduction in peak load level and reduction in voltage deviation are considered simultaneously as the objective functions. These objectives are first fuzzified and then integrated and introduced to a PSOTVAC/BFA
Algorithm in order to obtain the solution which minimizes the value of integrated objective function.

4. Developing adaptive membership functions: Fuzzy approach has been applied in previous works such as [16] for capacitor placement, but membership functions were predefined. This paper presents a method to find the appropriate membership functions in fuzzification process of objective functions. A method is also presented in order to make these objectives comparable with each other.

5. Improved economic modeling: Profit maximization is considered as one of the objective function while in order to justify the investment on DER installation comparing to the other investment opportunities, the Benefit to Cost Ratio (BCR) is considered as a constraint which its value should be greater than a predefined value. This predefined value should be calculated based on the other investment opportunities.

The proposed method is tested on IEEE-30 bus radial distribution test system. The simulation results show the effectiveness of the proposed method in DER planning problems and the necessity of stochastic modeling.

The rest of this paper is organized as follows. In section II an overview of the PSOTVAC/BFA algorithm is presented. The long term scenario generation and reduction procedures are described in section III. The proposed method is presented in section IV. The simulation results are presented and discussed in section IV. The conclusions are drawn section VI.

2. Hybrid PSOTVAC/BFA
A. Standard PSO
Classic PSO (CPSO) is one of the optimization techniques and a kind of evolutionary computation technique which is launched by the Aberhart Rasel. The method has been found to be robust in solving problems featuring nonlinearity and non-differentiability, multiple optima, and high dimensionality through adaptation, which is derived from the social-psychological theory. The features of the method are as follows [2]:
- The method is developed from research on swarm such as fish schooling and bird flocking.
- It is based on a simple concept. Therefore, the computation time is short and requires few memories [1].
- It was originally developed for nonlinear optimization problems with continuous variables. It is easily expanded to treat a problem with discrete variables.

CPSO is basically improved through simulation of bird flocking in two-dimension space. The position of each agent is defined by XY axis position and also the velocity is expressed by VX (the velocity of X axis) and VY (the velocity of Y axis). Modification of the agent position is notified by the position and velocity information. Bird flocking optimizes a certain objective function. Each agent knows its best value so far (pbest) and its XY position. This information is comparison of personal experiences of each agent. Moreover, each agent knows the best amount so far in the group (gbest). This information is comparison of knowledge of how the other agents around them have performed. Namely, each agent tries to update its position using the following information:
- The current positions (x, y),
- The current velocities (VX, VY),
- The distance between the current position and pbest

- The distance between the current position and gbest

This modification can be represented by the concept of velocity and the place of particle. Velocity of each agent can be modified by the following equation:

\[ V_i (t + 1) = \omega V_i (t) + c_1 r_1(t) [pbest_i (t) - x_i (t)] + c_2 r_2(t) [leader_i (t) - x_i (t)] \]  \hspace{1cm} (2)

Where,
- \( x_i \): position of agent i at iteration k
- \( v_i \): velocity of agent i at iteration k
- \( w \): inertia weighting
- \( c_{1,2} \): tilt coefficient
- \( r_{1,2} \): random number between 0 and 1
- \( \text{leader} \): archive of unconquerable particles
- \( \text{pbest} \): pbest of agent i
- \( \text{gbest} \): gbest of the group

Convergence of the PSO strongly depended on w, c1 and c2. While c1,2 are between 1.5 till 2, however the best choice to these factors is 2.05. Also, 0\leq w < 1; this value is really an important factor to the system convergence and it is better that this factor is defined dynamically. It should be between 0.2 and 0.9 and should decrease linear through evolution process of population. Being extra value of w at first, provides appropriate answers and small value of that help the algorithm to converge at the end.

B. PSO with Time-Varying Inertia Weight
The PSOTVIV method is capable of locating a good solution at a significantly faster rate, when compared with other meta-heuristic techniques; its ability to fine tune the optimum solution is comparatively weak, mainly due to the lack of diversity at the end of the search. Also, in PSO, problem-based tuning of parameters is a key factor to find the optimum solution accurately and efficiently. The main concept of PSOTVII is similar to CPSO in which the Eqs. (1), (2) are used. However, for PSOTVII the velocity update equation is modified by the constriction factor C and the inertia weight w is linearly decreasing as iteration grows.

\[ V_i (t + 1) = C \{ \omega V_i (t) + c_1 r_1(t) [pbest_i (t) - x_i (t)] + c_2 r_2(t) [leader_i (t) - x_i (t)] \} \]  \hspace{1cm} (3)

\[ \omega = (\omega_{\text{max}} - \omega_{\text{min}}) \frac{\left( k_{\text{max}} - k \right)}{k_{\text{max}}} + \omega_{\text{min}} \]  \hspace{1cm} (4)

\[ C = \frac{2}{\phi + \sqrt{4\phi^2 - 4\phi}} \text{where } 4.1 \leq \phi \leq 4.2 \]  \hspace{1cm} (5)

C. PSO with Time-Varying Acceleration Coefficients (PSO-TVAC)
Consequently, PSO-TVAC is extended from the PSOVTVAC. All coefficients including inertia weight and acceleration coefficients are varied with iterations. The equation of PSO-TVAC for velocity updating can be expressed as:
\[ V_i(t+1) = C \{ \omega_i(t) + ((c_{ij} - c_{ij}) \frac{k}{k_{max}} + c_{ij}) \}. \]

\[ r_i(t)[p_{best}(t) - x_i(t)] + ((c_{ij} - c_{ij2}) \frac{k}{k_{max}} + c_{ij2}) \cdot r_x(t)[leader_i(t) - x_i(t)]. \]

D. Bacteria Foraging Algorithm

Bacteria Foraging Algorithm (BFA) is one of the new optimization techniques which is based on the assumption that animals search for nutrients which maximizes their energy intake (E) per unit time (T) spent for foraging [3]. The E.coli bacterium is probably the best understood microorganism. Generally the bacteria move for a longer distance in a friendly environment.

Chemo-tactic Behavior of Escherichia Coli

We consider the foraging behavior of E. coli, which is a common type of bacteria. Its behavior and movement comes from a set of six rigid spinning (100–200 r.p.s) flagella, each driven as a biological motor. The E. coli bacterium alternates through running and tumbling. Running speed is 10–25 body lengths per second, however they can’t swim straight. The bacterium sometimes tumbles after a tumble or tumbles after a run [3]. This alternation between the two modes will move the bacterium, and this enables it to "search" for nutrients. If \( \theta(j,k,l) \) represent the position of the each member in the population of S bacterial at the jth chemotactic step, and \( l_{th} \) reproduction step, and \( l_{th} \) elimination, the movement of bacterium may be presented by:

\[ \theta(j+1,k,l) = \theta(j,k,l) + C(i) \phi(j) \]  \[ (7) \]

Where, \( C(i)(i = 1,2,\ldots,S) \) is the size of the step taken in the random direction specified by the tumble. \( \phi(j) \) is the random direction of movement after a tumble and \( J(i,j,k,l) \) is the fitness, which also denote the cost at the location of the \( l_{th} \) bacterium \( \theta(j,k,l) \) \( \in R^s \). Also if \( \theta(j+1,k,l) \) the cost \( J(i,j,k,l) \) is better (lower) than \( \theta(j,k,l) \), then another step of size \( C(i) \) in the same direction will be taken. Otherwise, bacteria will tumble via taking another step of size \( C(i) \) in random direction \( \phi(j) \) in order to seek better nutrient environment.

Swarming

An interesting group behavior has been observed for several motile species of bacteria including E.coli and S. typhimurium [8]. To achieve the function to model the cell-to-cell signaling with an attractant and a repellant. The E.coli swarming mathematical equation can be represented by:

\[ J_{cc}(\theta, P(j,k,l)) = \sum_{i=1}^{N} J_{cc}(\theta, \theta_i(j,k,l)) \]

\[ = \sum_{i=1}^{N} \left[ -d_{attract} \exp(-\omega_{attract} \sum_{m=1}^{n} (\theta_{m} - \theta_{m}^i)^2) \right] \]

\[ + \sum_{i=1}^{N} \left[ -r_{repellent} \exp(-\omega_{repellent} \sum_{m=1}^{n} (\theta_{m} - \theta_{m}^i)^2) \right] \]

The \( J_{cc}(\theta, P(j,k,l)) \) is the additional cost function added to the actual objective function (for minimization) to present a time varying objective function. The additional cost function \( J_{cc}(\theta, P(j,k,l)) \) for each bacterium \( \theta \) is composed of terms \( J_{cc}(\theta, \theta_i(j,k,l)) \) measuring attracting and repelling effects between two bacteria \( \theta \) and \( \theta_i \), illustrated in the next two lines of (5), respectively. In the original version of BF proposed by Passino [8], the parameters of \( d_{attract}, \omega_{attract}, \omega_{repellent} \) and \( \theta_i \) are set as follows:

\[ \omega_{attract}=0.2, \omega_{repellent}=10, d_{attract}=r_{repellent} \]

Considering the above parameters, each bacterium will try to move toward other bacteria to decrease the additional cost function \( J_{cc}(\theta, P(j,k,l)) \), but not too close to them, which is called swarming effect enhancing the local search capability of BFA. More details about (14) can be found in [8].

Reproduction

According to the rules of evolution, individual will reproduce themselves in appropriate conditions in a certain way. For bacterial, a reproduction step takes place after all chemotactic steps.

\[ J_{i health} = \sum_{j=1}^{N} J_I(i,j,k,l) \]

Where, \( J_{i health} = \text{health of bacterium i} \)

For keep a constant population size, bacteria with the highest \( J_{i health} \) values die. The remaining bacteria are allowed to split into two bacteria in the same place. Actually, in the reproduction loop only the poor individuals, which are unlikely to represent promising areas of the solution space, are filtered out and replaced by good solutions. In other words, the reproduction loop prevents wasting the search ability of BFA for searching non-promising areas of the solution space and thus the algorithm can concentrate on the promising areas of the solution space and search these areas with high accuracy and resolution. This characteristic leads to high local search ability of BFA. Moreover, different search paths are devised for the bacteria generated from the same individual in the next iterations of the loop, due to the chemotaxis operators, such as tumble and swim. In other words, the bacteria generated from the same individual will only be the same at the birth place, but will proceed in different directions and search the solution space through different paths. Consequently, the reproduction loop will not deteriorate the search diversity of BFA but can effectively enhance its search efficiency by filtering out poor individuals.
of the population and concentrating on the promising areas of the solution space.

Elimination-Dispersal
In evolutionary process, elimination and dispersal events can occur such that bacteria in a region are killed or a group is dispersed into a new part of the environment due to some influence. They have the effect of possibly destroying chemotactic progress, but they also have the effect of assisting in chemotaxis, since dispersal may place bacteria near good food sources. From the evolutionary point of view, elimination and dispersal was used to guarantee diversity of individuals and to strengthen the ability of global optimization [3]. In this technique to keeping the number of bacteria in the population constant, if a bacterium is eliminated, simply disperse one to a random location on the optimization domain.

E. Hybrid PSOTVAC-BFA
Actually, PSOTVAC is characterized as a simple, easy to implement and computationally efficient method, which is flexible with high global exploration ability. However, the local search ability of this algorithm is not as high as its global search ability and premature convergence may be occurred for the algorithm. In the opposite, the BFA algorithm via its adaptive reproduction and chemotaxis loop can effectively search promising areas of the solution space with high resolution enhancing the local search capability of PSOTVAC. However, there are some drawbacks in BFA in terms of its complexity and possibility to be locked up by a local solution. The proposed PSOTVAC can overcome these problems. Therefore, the algorithms have been combined such that each algorithm covers the deficiencies of the other one. The obtained hybrid method is designated as the hybrid PSOTVAC/BFA. The steps for executing the proposed hybrid method are:

STEP 1: Execute PSOTVAC as described.
STEP 2: Transport the solution obtained from the PSOTVAC to the BFA as an initial solution. The other initial individuals of the BFA are generated randomly within the allowable ranges.
STEP 3: Execute BFA as described.
STEP 4: Step 2 is run in the inverse direction such that the solution obtained by the BFA is transferred to the PSOTVAC and the initial population of the PSOTVAC is constructed.
STEP 5: Repeat steps 1-4 until the termination criterion is satisfied. Here, the termination criterion is set as the maximum number of iterations of the cycle 1-4.

3. Stochastic Long-term Model
In the proposed stochastic planning model, each possible system state is called scenario. These scenarios are created by the Monte Carlo simulation method to model long-term stochastic characteristics of the system components and bus loadings. Scenario reduction technique is used to decrease the number of created scenarios.

A. Monte Carlo simulation method
Fig. 1 depicts the annual load duration curve (LDC) that is modeled as multiple load blocks. This model is used as an infrastructure in consideration of forced outages of lines and load forecasting inaccuracies.

![Fig. 1. Load duration curve (LDC) of RTS.](image)

Hours with similar loads are shown in each load block in Fig. 1. Future annual peak load and energy demand growths are equal to the base year values times the regarding growth rates [15]. In this study the growth rate is expressed as an average growth rate, denoted by $AGRP$ and $AGRE$, and a random component, $RCP$, and $RCE$, for annual peak loads and total energy demands, respectively. Normally distributed random components with certain standard deviations are aggregated with the average growth rates to reflect the uncertainties in economic growth and/or weather changes. Random trajectories in 1st scenario and 9th year, represented by $P_{yr,s}$ for peak load, and represented by $E_{yr,s}$, for total energy demand, are expressed in the Monte Carlo simulation based on [18] as follows:

$$P_{yr,s} = P_{(yr-1),s} \times (1 + AGRP + RCP_{yr,s})$$  \hspace{0.5cm} (11)

$$E_{yr,s} = E_{(yr-1),s} \times (1 + AGRE + RCE_{yr,s})$$  \hspace{0.5cm} (12)

The future load block in scenario $s$ and year $yr$, denoted by $bl_{yr,s}$, is calculated via a linear transformation of the base year load and is formulated as

$$bl_{yr,s} = a_s \times b_0 + b_s$$  \hspace{0.5cm} (13)

Where

$$a_s = \frac{E_{yr,s} - 8760 \times P_{yr,s}}{E_0 - 8760 \times P_0}$$  \hspace{0.5cm} (14)

$$b_s = \frac{P_{yr,s} \times E_0 - P_0 \times E_{yr,s}}{E_0 - 8760 \times P_0}$$  \hspace{0.5cm} (15)

The bus load of bus $z$ at each load block is calculated by multiplying load distribution factor of that bus and load at each block.

$$PD_{xk,byr,s} = D_{xk,byr,s} \times bl_{yr,s}$$  \hspace{0.5cm} (16)

Transmission line availability of line $k$ at load block $b$ in year $yr$ denoted by $UY_{k,byr}$ is used in the Monte Carlo simulation in which $UY_{k,byr} = 1$ indicates that the transmission line $k$ is available at load block $b$ in year $yr$ while $UY_{k,byr} = 0$
indicates otherwise. Consequently, a scenario is consisted of \( RCP_{yr, s}, RCE_{yr, s} \) and \( UY_{yr, s, r} \).

B. Scenario Generation

This sub-section discusses generic scenarios generation. For each uncertain variable, different states are considered, each with a corresponding occurrence probability. Fig. 2 shows these states for system demand (as an example).

The sampling process in a non-sequential Monte Carlo is an iterative process, which is performed by generating a vector of random values between 0 and 1, each regarding to one of the uncertain variables. This vector is used to draw a sample, for example the value of demand (\( i \) th random variable) would be equal to \( \mu + st \cdot \sigma \) if (10) is fulfilled.

\[
\sum_{r=3}^{st-1} pr(r) < \text{rand}(i, sm) \leq \sum_{r=3}^{st} pr(r) \quad (10)
\]

![Probability density function of system demand.](image)

Modeling the load uncertainties using a normal distribution causes some errors and difficulties. In practical power systems, historical data on bus loads is available, so one can categorize the load historical samples at each bus in several groups based on their differences as a measure. The mean value of each group can be considered as different states. Occurrence probability of each group can be defined as the number of historical samples in this group divided by the total number of samples. However, this method of modeling cannot be used in our case studies, as a result of lack of historical data, so a normal distribution is applied to model the continues uncertainties.

There are different approaches for modeling the uncertainties associated with equipments’ unavailabilities. Assuming that each component has two states of failure and success, and components failures are independent of each other. If the random value is less than the failure rate, the component will be unavailable. This simple way of modeling considers multiple-outage modes and is almost sufficient in a non-sequential simulation. When a non-sever failure occurs, some components can still be operated in different derated states. These states are not modeled in this work.

Probability of each sample can be calculated using (17).

\[
prob(sm) = \prod_{r=1}^{R} \text{rand}(sm, r) \quad (17)
\]

C. Scenario reduction

The computational requirements for solving scenario-based optimization models are directly affected by the number of scenarios. Therefore an effective scenario reduction technique could be very essential and useful in solving large-scale problems. Reference [18] defines reduction technique as a scenario-based approximation with a smaller number of scenarios and a reasonably good approximation of the original system. The scenario reduction technique that is applied in this study control the goodness-of-fit of approximation by measuring a distance of probability distributions as a probability metric. After performing scenario reduction, a subset of scenarios with regarding probabilities is selected that models the initial probability distribution in terms of probability metrics. Efficient algorithms based on backward and fast forward methods are derived that determine optimal reduced measures. An overview of the simultaneous backward reduction method, based on [25], is given in the following.

Let \( \xi_s (s=1, ..., Sc) \) denote \( Sc \) different scenarios, each with a probability of \( Prob_\xi \), and \( Dist_{xy} \) be the distance of scenario pair \( (s, s') \). The simultaneous backward and fast forward is given in the following steps:

Step 1: Set \( S \) as the initial set of scenarios; \( DS \) is the set of scenarios to be omitted. The initial \( DS \) is null. Compute the distances of all scenario pairs:

\[
Dist_{xy} = \text{Dist}(\xi_s, \xi_s'), s, s' = 1, ..., Sc;
\]

Step 2: for each scenario \( k \), \( Dist_k(r) = \text{min} Dist_{xy} \), while \( s' \neq k \), \( r \) is the index of scenario that has the minimum distance with scenario \( k \);

Step 3: compute \( DD_k(r) = Prob_\xi \times Dist_k(r) \), \( k \in S \). Choose \( d \) so that \( DD_k = \text{min} (DD_k), k \in S \);

Step 4: \( S = S - \{d\}, DS = DS + \{d\}; Prob_\xi = Prob_\xi + Prob_\xi \);

Step 5: repeat steps 2-4 until the number to be deleted meets the predefined number of scenarios.

4. Proposed Method

The aim of operation and planning in deregulated power systems is to maximize the social welfare through minimization of costs of the network, while the electric power is delivered to the customers with sufficient quality and reliability. Because of the high investment cost of DERs, there is considerable risk in their application. Therefore the optimal placement and sizing of DERs are the most important steps to be performed considering various aspects of distribution networks. The objectives of this study are loss minimization, reduction of power which should be purchased from electricity market, loss reduction at the peak load level and improvement of voltage profile of the power system through proper application of DERs.
A. Objective Fuzzification

Each objective in fuzzy domain is associated with a membership function. The membership function specifies the degree of satisfaction of the objective. In the crisp domain, the objective is either satisfied or violated, indicating membership values of unity and zero, respectively. On the contrary, fuzzy sets consider varying degrees of membership function values from zero to unity [16]. The present work considers the following objectives for the DER placement problem:

- Maximization of the saving by minimization of the energy loss, power purchased and loss at the peak load level due to the application of DERs.
- Minimization of the voltage deviation at network buses.

Before continuing further in this section let us return to the stochastic long term load model and find out how one can use it for DER placement.

In each scenario we have 12 load levels representing four different load blocks of the three years of study horizon, each with a chance of occurrence. Each scenario itself has a probability. Combining the load levels of the scenarios, the total load Probability Density Function (PDF) is obtained. The load PDF is divided into equal sections each with an occurrence probability. The centers of these sections are introduced to the optimization algorithm as the load levels.

The membership function consists of a lower and upper bound values along with a strictly monotonically decreasing and continuous function are described in the following.

B. Membership function for the net saving

The net saving at 4th load level due to application of DER in a distribution system is given as the following (it should be noted that load levels are in fact the stochastic load levels that along with line outages reflect the system states in each scenario):

\[
N_s = \sum_{i=1}^{N_{DER}} K_{i}^{\text{DER}} \rho_i \sum_{t=1}^{N_{yr}} P_{t,i}^{\text{DER}} - \sum_{i=1}^{N_{DER}} K_{i}^{\text{DER}} P_{i,\text{Max}}
\]

(18)

where, \(K_{i}^{\text{DER}}\) is a factor to convert peak power loss reduction to dollar (S/kwh); \(\rho_i\) is the duration of peak load (hours); \(P_{t,i}^{\text{DER}}\) is the power loss reduction at peak load level at year \(yr\) due to application of DERs (kwh); \(K_{i}^{\text{DER}}\) is a factor to convert energy losses to dollar (S/kwh); \(\rho_i\) is the probability of ith load level; \(L_{i}^{\text{DER}}\) is the reduction in power loss at kth load level due to application of DERs (kwh); \(N_{DER}\) is the number of DERs; \(P_{i,\text{Max}}^{\text{DER}}\) is the power output of ith DER at kth load level (kwh); \(K_{i}^{\text{DER}}\) is the cost of operation and maintenance of DER at kth load level (S/kwh); \(K_{i}^{\text{DER}}\) is the cost of investment of DER (S/kwh); and \(P_{i,\text{Max}}^{\text{DER}}\) is the maximum capacity of ith DER (kwh).

Considering a positive profit for application of DERs, for net saving in (13), we have \(N_s > 0\) that means:

\[
\sum_{i=1}^{N_{yr}} [K_{i}^{\text{DER}} LR_{i}^{\text{peak}} + K_{i}^{\text{DER}} \rho_i \sum_{t=1}^{N_{yr}} P_{t,i}^{\text{DER}}] + \sum_{i=1}^{N_{yr}} K_{i}^{\text{DER}} P_{i,\text{Max}}^{\text{DER}}
\]

(19)

\[
\sum_{i=1}^{N_{yr}} [K_{i}^{\text{DER}} LR_{i}^{\text{peak}} + K_{i}^{\text{DER}} \rho_i \sum_{t=1}^{N_{yr}} P_{t,i}^{\text{DER}}] + \sum_{i=1}^{N_{yr}} K_{i}^{\text{DER}} P_{i,\text{Max}}^{\text{DER}} \geq 0
\]

(20)

Let us define

\[
x_{k} = \frac{\sum_{i=1}^{N_{yr}} [K_{i}^{\text{DER}} LR_{i}^{\text{peak}} + K_{i}^{\text{DER}} \rho_i \sum_{t=1}^{N_{yr}} P_{t,i}^{\text{DER}}] + \sum_{i=1}^{N_{yr}} K_{i}^{\text{DER}} P_{i,\text{Max}}^{\text{DER}}}{\sum_{i=1}^{N_{yr}} [K_{i}^{\text{DER}} LR_{i}^{\text{peak}} + K_{i}^{\text{DER}} \rho_i \sum_{t=1}^{N_{yr}} P_{t,i}^{\text{DER}}] + \sum_{i=1}^{N_{yr}} K_{i}^{\text{DER}} P_{i,\text{Max}}^{\text{DER}}}
\]

(21)

Eq. (15) indicates that if \(x_{k}\) is high, the net saving (profit) is low and vice versa. Membership function for the net saving (profit) is given in Fig. 3. Based on this figure one can reach the following equations:

\[
\mu_{S_k} = \frac{(x_{\max} - x_k)}{(x_{\max} - x_{\min})} \text{ for } x_{\min} \leq x_k \leq x_{\max}
\]

(22)

\[
\mu_{S_k} = 1 \text{ for } x_k \leq x_{\min}
\]

(23)

\[
\mu_{S_k} = 0 \text{ for } x_k \geq x_{\max}
\]

(18)

In this study, \(x_{\max}\) is assumed to be 1.0; in order to achieve \(x_{\max}\), the proposed method is once performed without consideration of voltage improvement as one of the objectives. The value of \(x_{\max}\) of 0.375 means unity membership value is assigned if the savings is 37.5% or more, and the \(x_{\max}\) of 1.0 means zero membership value is assigned if the profit is zero percent of the cost or has a negative value.
C. Membership function for the node voltage deviation

Basic purpose of this membership function is that the deviation of nodes voltage should be minimized. At $k$th load level of the load duration curve, let us define $y_k = \max(\text{abs}(V_{n,i} - V_{i,k})) \quad \text{for} \quad i = 2,3,...,N_v$ (24)

Where, $V_{i,k}$ is the voltage magnitude of node $i$ at $k$th load level in per unit and $V_n$ is the nominal voltage magnitude that is equal to one in per unit. It should be noted that it is for the case that the substation is located at bus 1. The less the maximum value of nodes voltage deviation, the high the assigned membership value and vice versa. Fig. 4 shows the membership function for maximum node voltage deviation defined in [16]. Based on Fig. 3 and taking into account $y_{min} \leq y_k \leq y_{max}$ we have:

$$\mu_{y_k} = \begin{cases} (\frac{y_k - y_{max}}{y_{max} - y_{min}}) & y_k \geq 0 \\ (\frac{y_k - y_{min}}{y_{max} - y_{min}}) & y_k \leq 0 \end{cases}$$ (25)

$$\mu_{y_k} = 1 \quad \text{for} \quad y_{min} \leq y_k \leq y_{max}$$ (26)

$$\mu_{y_k} = 0 \quad \text{for} \quad y_k \geq y_{max} \text{ or } y_k \leq y_{min}$$ (27)

Fig. 4. Membership function of node voltage deviation.

In this study, $y_{min}$ and $y_{max}$ are considered to be 0.05 and 0.10, respectively. $y_{min}$ and $y_{max}$ are assumed -0.05 and -0.10, respectively. Considering $V_n = 1$, $y_{max} = 0.05$ means the minimum system voltage will be 0.95 p.u. and it means that if the minimum system voltage is greater than or equal to 0.95 p.u., the membership value is one. Similarly, $y_{max} = 0.10$ means the minimum system voltage allowed will be 0.90 p.u. and if the minimum system voltage is less than or equal to 0.90 p.u., the assigned membership value will be zero. For more information about practical issues about how the membership functions and weighting factors should be chosen please refer to [12]-[14].

D. Fuzzy Formulation for Several Objectives

The two fuzzified objectives described in the previous section are dealt with by integrating them into a fuzzy satisfaction objective function $F$, through appropriate weighting factors $(K)$ as (23).

$$\text{Max} \quad F = K_1 \times \mu_{s_1} + K_2 \times \mu_{s_2}$$ (28)

$K_1$ and $K_2$ are weighting factor considered in this study for investigation of the impact of each one of objectives in planning of the DERs. $K_1 = 0$ and $K_2 = 1$ means that only improving voltage profile is considered as the objective of optimization and vice versa; while $K_1 = 0.5$ and $K_2 = 0.5$ means that these two objectives are assumed to be equally important. The weighting factors can be determined according to the preferences of the operators [16].

5. Simulation Results

In order to test the effectiveness of the proposed method, IEEE 30-bus distribution system including 22 load points, 6 auxiliary substations and a main feeder [26] is chosen. This system is shown in Fig. 5.

The load pattern in the peak load level of the present year is shown in Table I. Both real ($kW$) and reactive ($kVAR$) loads are specified. The base value of voltage and power are 23 (kV) and 100 (MVA), respectively. Table I also shows the resistance and reactance of the lines. After scenario aggregation ten load levels are considered, each with an occurrence probability. These load levels are presented in Table II as the percentage of the peak load level of the present year. One can consider more load levels for the sake of accuracy. The PSO parameters are presented in Table III.

Fig. 5. IEEE 30-bus distribution system
TABLE I
IEEE 30-BUS DISTRIBUTION SYSTEM DATA

<table>
<thead>
<tr>
<th>From Bus i</th>
<th>To Bus j</th>
<th>Active load at j (MW)</th>
<th>Reactive load at j (MVAR)</th>
<th>r_j (pu)</th>
<th>x_j (pu)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Feeder</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0.0963</td>
<td>0.3219</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.5220</td>
<td>0.1740</td>
<td>0.0414</td>
<td>0.0022</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0.0659</td>
<td>0.0651</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>0.9360</td>
<td>0.3120</td>
<td>0.2221</td>
<td>0.1931</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0</td>
<td>0</td>
<td>0.1045</td>
<td>0.0909</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0</td>
<td>0</td>
<td>0.3143</td>
<td>0.1770</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0.2553</td>
<td>0.1438</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>0.1890</td>
<td>0.0630</td>
<td>0.2506</td>
<td>0.1412</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>0.2506</td>
<td>0.1412</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>0.3360</td>
<td>0.1120</td>
<td>0.7506</td>
<td>0.4229</td>
</tr>
<tr>
<td></td>
<td>11</td>
<td>0.6570</td>
<td>0.2190</td>
<td>0.3506</td>
<td>0.1975</td>
</tr>
<tr>
<td></td>
<td>12</td>
<td>0.7830</td>
<td>0.2610</td>
<td>0.1429</td>
<td>0.0805</td>
</tr>
<tr>
<td></td>
<td>13</td>
<td>0.7290</td>
<td>0.2430</td>
<td>0.2909</td>
<td>0.1639</td>
</tr>
<tr>
<td></td>
<td>14</td>
<td>0.4770</td>
<td>0.1590</td>
<td>0.0898</td>
<td>0.0781</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>0.5490</td>
<td>0.1830</td>
<td>0.1377</td>
<td>0.0775</td>
</tr>
<tr>
<td></td>
<td>16</td>
<td>0.4770</td>
<td>0.1590</td>
<td>0.2467</td>
<td>0.1390</td>
</tr>
<tr>
<td></td>
<td>17</td>
<td>0.4320</td>
<td>0.1440</td>
<td>0.0915</td>
<td>0.0795</td>
</tr>
<tr>
<td></td>
<td>18</td>
<td>0.6720</td>
<td>0.2240</td>
<td>0.3005</td>
<td>0.2612</td>
</tr>
<tr>
<td></td>
<td>19</td>
<td>0.4950</td>
<td>0.1650</td>
<td>0.2909</td>
<td>0.1639</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>0.2070</td>
<td>0.0690</td>
<td>0.1143</td>
<td>0.0994</td>
</tr>
<tr>
<td></td>
<td>21</td>
<td>0.5220</td>
<td>0.1740</td>
<td>0.1066</td>
<td>0.1054</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>1.9170</td>
<td>0.0630</td>
<td>0.0649</td>
<td>0.0641</td>
</tr>
<tr>
<td></td>
<td>23</td>
<td>0</td>
<td>0</td>
<td>0.1083</td>
<td>0.0941</td>
</tr>
<tr>
<td></td>
<td>24</td>
<td>1.1160</td>
<td>0.3720</td>
<td>0.2760</td>
<td>0.2399</td>
</tr>
<tr>
<td></td>
<td>25</td>
<td>0.5490</td>
<td>0.1830</td>
<td>0.2009</td>
<td>0.1746</td>
</tr>
<tr>
<td></td>
<td>26</td>
<td>0.7920</td>
<td>0.2640</td>
<td>0.2857</td>
<td>0.1609</td>
</tr>
<tr>
<td></td>
<td>27</td>
<td>0.8820</td>
<td>0.2940</td>
<td>0.0881</td>
<td>0.0047</td>
</tr>
<tr>
<td></td>
<td>28</td>
<td>0.8820</td>
<td>0.2940</td>
<td>0.3091</td>
<td>0.1741</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>0.8820</td>
<td>0.2940</td>
<td>0.2106</td>
<td>0.1187</td>
</tr>
</tbody>
</table>

TABLE II
QUANTIZED LOAD LEVELS AND THEIR RESPECTING PROBABILITY

<table>
<thead>
<tr>
<th>Load</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.626</td>
</tr>
<tr>
<td>2</td>
<td>0.693</td>
</tr>
<tr>
<td>3</td>
<td>0.759</td>
</tr>
<tr>
<td>4</td>
<td>0.826</td>
</tr>
<tr>
<td>5</td>
<td>0.892</td>
</tr>
<tr>
<td>6</td>
<td>0.958</td>
</tr>
<tr>
<td>7</td>
<td>1.025</td>
</tr>
<tr>
<td>8</td>
<td>1.091</td>
</tr>
<tr>
<td>9</td>
<td>1.157</td>
</tr>
<tr>
<td>10</td>
<td>1.224</td>
</tr>
</tbody>
</table>

It is assumed that the size of DERs varies in 100 (kW) steps. The investment and operation costs of DERs are borrowed from [27].

TABLE III
PSO ALGORITHMS’ PARAMETERS

<table>
<thead>
<tr>
<th>Swarm Size</th>
<th>Deterministic Problem</th>
<th>Probabilistic Problem</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C1</td>
<td>C2</td>
</tr>
<tr>
<td>30</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>50</td>
<td>1.7</td>
<td>1.7</td>
</tr>
</tbody>
</table>
A. Fuzzy Optimization Problem Considering Several Objectives, Deterministic case

Before testing the proposed stochastic method, a deterministic version of the proposed method is tested on IEEE 30-bus distribution system in this sub-section. The solution of this deterministic problem can be compared with the solution of stochastic problem to show the necessity of the stochastic modeling of the problem. The values of the load and energy growth rates are considered to be 0.08. The load model with nine levels is used for the three-year time horizon. Table IV shows these load levels and the regarding time durations. The proposed stochastic approach can be simply modified for deterministic problem by substituting the probability of load levels (ρk) with load level durations of Table IV.

<table>
<thead>
<tr>
<th>Load level</th>
<th>Duration in 3-year (hrs)</th>
<th>Duration in 3-year (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.500</td>
<td>2000</td>
</tr>
<tr>
<td>2</td>
<td>0.540</td>
<td>2000</td>
</tr>
<tr>
<td>3</td>
<td>0.583</td>
<td>2000</td>
</tr>
<tr>
<td>4</td>
<td>0.700</td>
<td>5260</td>
</tr>
<tr>
<td>5</td>
<td>0.756</td>
<td>5260</td>
</tr>
<tr>
<td>6</td>
<td>0.816</td>
<td>5260</td>
</tr>
<tr>
<td>7</td>
<td>1.000</td>
<td>1500</td>
</tr>
<tr>
<td>8</td>
<td>1.080</td>
<td>1500</td>
</tr>
<tr>
<td>9</td>
<td>1.166</td>
<td>1500</td>
</tr>
</tbody>
</table>

In order to find the maximum attainable profit which as discussed earlier is an important factor in construction of membership functions, firstly the voltage deviation is omitted from the objective function and the maximum profit found is $1389923. Now we can find the suitable membership function regarding to profits and solve the problem. Table V shows the optimal solution of the deterministic problem.

Table V

<table>
<thead>
<tr>
<th>Bus</th>
<th>Size of DER (KW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>600</td>
</tr>
<tr>
<td>11</td>
<td>1500</td>
</tr>
<tr>
<td>14</td>
<td>300</td>
</tr>
<tr>
<td>15</td>
<td>300</td>
</tr>
<tr>
<td>22</td>
<td>900</td>
</tr>
<tr>
<td>25</td>
<td>1700</td>
</tr>
</tbody>
</table>

The value of profit is still acceptable comparing with the maximum attainable profit gained in previous case study.

B. Fuzzy Optimization Problem Considering Several Objectives, Stochastic case

In this case study the proposed stochastic approach is used to find the best solution of the optimization problem considering several objectives. In order to find the shape of membership function of the first part of the objective function, initially the stochastic problem is solved considering the profit as the objective function to find the maximum attainable value of profit. Table VI show the results of placement problem for the single objective problem. As can be seen in this table the maximum attainable profit with the minimum acceptable BCR is $6658313.1. At the next stage the stochastic problem is solved considering all the objectives and the results are shown in Table VII.

Table VI

<table>
<thead>
<tr>
<th>Bus</th>
<th>Size of DER (KW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>13</td>
<td>1900</td>
</tr>
<tr>
<td>17</td>
<td>1700</td>
</tr>
<tr>
<td>19</td>
<td>1000</td>
</tr>
<tr>
<td>21</td>
<td>1000</td>
</tr>
<tr>
<td>24</td>
<td>2800</td>
</tr>
</tbody>
</table>

Table VII

<table>
<thead>
<tr>
<th>Bus</th>
<th>Size of DER (KW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1900</td>
</tr>
<tr>
<td>14</td>
<td>1400</td>
</tr>
<tr>
<td>15</td>
<td>1800</td>
</tr>
<tr>
<td>17</td>
<td>400</td>
</tr>
<tr>
<td>24</td>
<td>500</td>
</tr>
<tr>
<td>26</td>
<td>500</td>
</tr>
<tr>
<td>27</td>
<td>1400</td>
</tr>
</tbody>
</table>

Fig. 7 shows convergence characteristic for the stochastic problem by PSOTVAC/BFA. This figure depicts the change in the BCR of the best solution (Queen) versus iterations of the algorithm. As it can be seen, the PSOTVAC/BFA have rapid convergence characteristic.
C. Statistical analysis of the results

Since the algorithm used here is a heuristic optimization algorithm and the stochastic nature of the system has been taken into account, the results derived from the proposed method might vary in each run. In order to investigate the effect of these factors, statistical analysis of the results is discussed in this subsection. The proposed algorithm is run 50 times to find the best solution of the problem discussed in case study B (stochastic case with both objectives included).

<table>
<thead>
<tr>
<th>TABLE VIII</th>
<th>STATISTICAL ANALYSIS OF THE RESULTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-year Profit ($)</td>
<td>Best solution</td>
</tr>
<tr>
<td>6255752.6</td>
<td>6256712.1</td>
</tr>
<tr>
<td>LR (MWh)</td>
<td>3671.32</td>
</tr>
<tr>
<td>LRP (KWh)</td>
<td>942.2</td>
</tr>
<tr>
<td>PPR (MWh)</td>
<td>197015</td>
</tr>
</tbody>
</table>

Table VIII shows the results of statistical. The PSOTVAC/BFA parameters are considered to be fixed in all runs. As can be seen the standard deviation of the solutions is very low. This shows the robustness of the proposed algorithm against some factors such as initial population of PSOTVAC/BFA algorithm. Another study is also conducted to analyze the effects of increase in degree of uncertainty associated with system loads. The standard deviation of peak load and energy growth is changed from 2 to 5% and the results are presented in Table IX. As can be seen in this table the value of the profit decreases as the degree of uncertainty increases. It is also interesting that as the degree of uncertainty increases, the maximum value of voltage deviation increases with one exception. The reason may lay under this fact that with increase in these standard deviations the total objective function will definitely decrease, but each objective may show unexpected trend.

<table>
<thead>
<tr>
<th>TABLE IX</th>
<th>EFFECTS OF UNCERTAINTIES ON THE OPTIMAL SOLUTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-year Profit ($)</td>
<td>$\sigma = 2%$</td>
</tr>
<tr>
<td>6255752.6</td>
<td>6255402.5</td>
</tr>
<tr>
<td>Maximum voltage deviation (pu)</td>
<td>0.0412</td>
</tr>
</tbody>
</table>

D. Discussion

In order to show the effectiveness of the proposed method this section provides a discussion about the results of case studies.

1. As said earlier the proposed algorithm schedules the output of DERs in each load level individually to avoid the unwanted rejection of optimal solutions. To illustrate this, consider the result of Table VII. The reduction in power purchased (PPR) from the electricity market simply shows that in all load levels the maximum output of DERs is not scheduled. The maximum power which can be supplied by DERs is 7900 kW, so the maximum energy which can be supplied is $3 \times 8760 \times 7.9 = 207612$ (MWh) while the energy served by DERs is 197015 (MWh). Again the minimum BCR of 1.3 is considered as a constraint.

2. Comparing the results of Table VII with those presented in Table VI, the profit and the maximum voltage deviation are less for the problem in which several objectives are considered. This shows that the improvement of voltage profile causes a small reduction in profit.

3. Comparing the results of stochastic algorithm (Table VII) to those obtained in case study (section V.I) for deterministic problem, one can understand the necessity of the stochastic modeling of the problem. In order to clarify this point the solution of deterministic problem is fed into the stochastic model to calculate the stochastic profit gained by applying best solution of deterministic problem. The profit value is 441295.2 ($\)$ which is so much lower than the stochastic profit gained by best solution of stochastic problem reported in Table VII (6255752.6 ($\)$).

4. The voltage profile in the peak load level of each year of the three-year time horizon of the study for compensated and uncompensated cases are presented in Fig. 8. As can be seen in this figure the proposed method can effectively mitigate the voltage deviation. It should be noted that the optimal power flow algorithm for peak load level of years 2 and 3 in uncompensated case did not converged with the main feeder voltage of 1.05 (p.u.); so the voltage of main feeder for these states is considered to be 1.1 (p.u.). Table X shows the mean deviation of voltage at different load points and in fact summarized the results presented in Fig. 8.

<table>
<thead>
<tr>
<th>TABLE X</th>
<th>MEAN VALUE OF VOLTAGE DEVIATION AT PEAK LOAD LEVEL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Without DER</td>
<td>First year</td>
</tr>
<tr>
<td>0.0350</td>
<td>0.0378</td>
</tr>
<tr>
<td>With DER</td>
<td>0.028502</td>
</tr>
</tbody>
</table>
6. Conclusion

A PSO based fuzzy stochastic long term optimization methodology considering several objectives has been proposed in this paper for optimal placement and sizing of DERs. As the results of case studies shows, ignoring the uncertainties in DER placement problem renders the stochastic saving gained non-optimal.

![Voltage profile at peak load level for compensated and uncompensated systems at the peak load level of the present year, deterministic case.](image1)

![Voltage profile at peak load level of each year for compensated and uncompensated systems, stochastic case](image2)

The optimization algorithm simultaneously seeks the reduction in power loss; power purchased from the electricity market, power loss at the peak load level and deviation of the voltage magnitude at the load points. A proper modeling of economic aspects of the problem is also presented in this paper. The results of case studies show that the proposed algorithm can effectively guarantee the justification of investment on DERs. The results also show that the method schedules the output power of DERs in each load level and avoids the inconvenience rejection of more optimal solutions. In fuzzifying process the membership functions have been obtained with an effective method, instead of the predefined membership functions. Some other aspects of the proposed method are discussed in case studies.

REFERENCES

1. A. M. El-Zonkoly, A. A. Khalil and N. M. Ahmied, “Optimal tuning of lead-lag and fuzzy logic power system